

Hicks Type Contractions and SB-type Contractions in Fuzzy Metric Spaces

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Abstract- In this paper, we give a generalization of Hicks type contractions and SB- type contractions in fuzzy metric spaces. We prove some fixed point theorems for these new type contraction mappings on fuzzy metric spaces. Moreover, we compare between of these of contractions. These results generalize some known results in fuzzy metric spaces and probabilistic metric spaces.

Keywords: Contraction mappings, Fixed point theorems, Hicks type contraction, SB-type contraction.

I. INTRODUCTION

In 1972, the notion of contraction mappings on probabilistic metric space was introduced by Sehgal and Bharucha [1] and they proved that every such mapping on a complete Menger probabilistic metric space (X, F, Δ) has a unique fixed point. Subsequently, Sherwood [2] showed that a very large class of t -norms it is possible to construct complete Menger probabilistic metric space together with contraction mapping which have no fixed point. Recently, Hicks [3] considered another notion of contraction mappings and he showed that every such mapping on a complete Menger probabilistic metric space (X, F, Δ) has a unique fixed point. In this paper, we give the new version of Hicks type contraction in fuzzy metric space.

In this paper, we shall give a generalization of Hicks type contractions and SB-type contractions on fuzzy metric spaces and prove some fixed point theorems for this new type contraction mappings on fuzzy metric spaces. Let \mathbb{N} be the set of all positive integers. The structure of this paper is as follows. In Section 2, we recall some definitions and the uniform structure of fuzzy metric spaces. In Section 3, we give some concepts on SB-contraction and prove some fixed point theorems in fuzzy metric spaces. Section 4 is devoted to introduce a version of Hicks-contraction in fuzzy metric space. In section 5, we compare between of these two contractions. Our results generalize and extend many known results in fuzzy metric spaces and probabilistic metric spaces, see [4-6].

II. PRELIMINARIES

In this section, some definitions and preliminary results are given which will be used in the sequel.

Definition 2.1 [7]. A binary operation $\Delta: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t -norm if Δ satisfying the following conditions:

- (1) Δ is commutative and associative;
- (2) Δ is continuous;
- (3) $\Delta(a; 1) = a$ for all $a \in [0; 1]$;
- (4) $\Delta(a; b) \leq \Delta(c; d)$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0; 1]$.

Some typical examples of t -norm are the following:

$$\Delta(a; b) = ab; \quad (\text{product})$$

$$\Delta(a; b) = \min\{a, b\}; \quad (\text{minimum})$$

$$\Delta(a; b) = \max\{a + b - 1, 0\}; \quad (\text{Lukasiewicz})$$

$$\Delta(a, b) = \frac{ab}{a+b-ab}; \quad (\text{Hamacher})$$

Definition 2.2 [5]. A triple (X, M, Δ) is called a fuzzy metric space (briefly, a FM-space) if X is an arbitrary (non-empty) set, Δ is a continuous t -norm and M is a fuzzy set on $X \times X \times [0, 1)$ such that the following axioms hold:

(FM-1) $M(x, y, 0) = 0$ for all $x, y \in X$,

(FM-2) $M(x, y, t) = 1$ for every $t > 0$ if and only if $x = y$,

(FM-3) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$,

(FM-4) $M(x, y; \cdot) : [0; \infty) \rightarrow [0; 1]$ is left continuous for all $x, y \in X$,

(FM-5) $M(x, z, t + s) \geq \Delta(M(x, y, t), M(y, z, s))$ for all $x, y, z \in X$ and for all $t, s \in [0, \infty)$.

We will refer to the fuzzy metric spaces in the sense of Kramosil and Michalek as KM-fuzzy metric spaces. If, in the above definition, the condition (FM-5) is replaced by the condition:

(FM-5A) $M(x, z, \max\{t, s\}) \geq \Delta(M(x, y, t), M(y, z, s))$ for all $x, y, z \in X$ and for all $t, s \in [0, \infty)$.

Then (X, M, Δ) is called a strong metric space. It is easy to check that (FM-5A) implies (FM-5), that is, every strong fuzzy metric space is itself a fuzzy metric space.

Definition 2.3 [9]. A triple (X, M, Δ) is called a fuzzy metric space (briefly, a FM-space) if X is an arbitrary (non-empty) set, Δ is a continuous t -norm and M is a fuzzy set on $X \times X \times [0, 1)$ such that the following axioms hold:

(FM-1) $M(x, y, t) > 0$ for all $x, y \in X$ and $t > 0$,

(FM-2) $M(x, y, t) = 1$ for every $t > 0$ if and only if $x = y$,

(FM-3) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$,

(FM-4) $M(x, y; \cdot) : [0; \infty) \rightarrow [0; 1]$ is left continuous for all $x, y \in X$,

(FM-5) $M(x, z, t + s) \geq \Delta(M(x, y, t), M(y, z, s))$ for all $x, y, z \in X$ and for all $t, s \in [0, \infty)$.

We will refer to the fuzzy metric spaces in the sense of George and Veeramani as GV-fuzzy metric spaces.

Example 2.1 [9]. (1) Let (X, d) be a metric space. Define a t -norm by $\Delta(a, b) = ab$, and set

$$M_d(x, y, t) = \frac{t}{t + d(x, y)};$$

for all $x, y \in X$ and $t > 0$.

Then (X, M_d, Δ) is a strong fuzzy metric space; M_d is called the standard fuzzy metric induced by d . It is interesting to note that the topology induced by the M_d and the corresponding metric d coincide.

(2) Let (X, d) be a metric space. Define a t -norm by $\Delta(a, b) = ab$ and set

$$M(x, y, t) = \exp\left(-\left[\frac{d(x, y)}{t}\right]\right), \quad \text{for all } x, y \in X \text{ and } t > 0.$$

Then (X, M, Δ) is a strong fuzzy metric space.

Lemma 2.1 [8]. Let (X, M, Δ) be a FM-spaces. Then $M(x, y, t)$ is non-decreasing with respect to t , for all $x, y \in X$.

Lemma 2.2 [11]. Let (X, M, Δ) be a FM-spaces. If there exists $k \in (0, 1)$ such that

$$M(x, y, kt) \geq M(x, y, t)$$

for all $x, y \in X$ and $t > 0$, then $x = y$.

Definition 2.4 [9]. Let (X, M, Δ) be a fuzzy metric space with a continuous t -norm Δ .

(1) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for every $\varepsilon > 0$ and

$\lambda > 0$ there exists a positive integer N such that $M(x_n, x, \varepsilon) > 1 - \lambda$ whenever $n \geq N$;

(2) A sequence $\{x_n\}$ in X is said to be Cauchy sequence in X if for every $\varepsilon > 0$ and

$\lambda > 0$ there exists a positive integer N such that $M(x_n, x_m, \varepsilon) > 1 - \lambda$ whenever $n, m \geq N$;

(3) (X, M, Δ) is complete if every Cauchy sequence in X is convergent to some point in X .

The (ε, λ) -topology τ in a fuzzy metric space is introduced by the family of neighborhoods N_x of a point $x \in X$ given by

$$N_x = \{N_x(\varepsilon, \lambda) : \varepsilon > 0 \text{ and } \lambda \in (0, 1)\}$$

where

$$N_x(\varepsilon, \lambda) = \{y \in X : M(x, y, \varepsilon) > 1 - \lambda\}.$$

The (ε, λ) -topology τ is a Hausdorff and first countable. In this topology the function T is continuous at $x \in X$ if and only if for every sequence $x_n \rightarrow x$ it holds that $Tx_n \rightarrow Tx$.

III. SB-CONTRACTION MAPPINGS IN FM-SPACES

In this section, we give some fixed point theorems of the SB-contraction type in fuzzy metric spaces.

Definition 3.1. Let (X, M, Δ) be a fuzzy metric space with continuous t -norm Δ . A mapping $T : X \rightarrow X$ is a contraction mapping (or a SB-contraction mapping) if and only if there is an $\mu \in (0, 1)$ such that

$$M(Tx, Ty, t) \geq M\left(x, y, \frac{t}{\mu}\right), \text{ for every } x, y \in X \text{ and } t > 0. \quad (1)$$

Lemma 3.2. Let (X, M, Δ) be a fuzzy metric space with continuous t -norm Δ . Let $T : X \rightarrow X$ be a contraction mapping satisfying condition (1). Then either

- 1) T has a unique fixed point, or
- 2) For every $p_0 \in X$, $\sup\{G_{p_0}(t) : t \geq 0\} < 1$, where

$$G_{p_0}(t) = \inf_n \{M(p_0, p_m, t) : p_m = Tp_{m-1}, m \in \mathbb{N}\}.$$

Proof. Suppose that there exists a point $p_0 \in X$ such that $\sup\{G_{p_0}(t): t \geq 0\} = 1$. Then we have

$$M(p_n, p_{n+m}, t) \geq M(p_0, p_m, \frac{t}{\mu^n}) \geq G_{p_0}\left(\frac{t}{\mu^n}\right).$$

Thus, since G_{p_0} is non-decreasing,

$$\lim_{n \rightarrow \infty} M(p_n, p_{n+m}, t) = 1$$

for all $t > 0$ independent of m , i.e., $\{p_n\}$ is a Cauchy sequence in a τ -complete fuzzy metric space. Consequently, there is a point $p^* \in X$ such that $\{p_n\}$ converges to p^* . To see that $Tp^* = p^*$ it suffices to notice that, for every positive integer n ,

$$\begin{aligned} M(Tp^*, p^*, t) &\geq \Delta\left(M\left(Tp^*, p_n, \frac{t}{2}\right), M\left(p^*, p_n, \frac{t}{2}\right)\right) \\ &\geq \Delta\left(M\left(Tp^*, p_{n-1}, \frac{t}{2}\right), M\left(p^*, p_n, \frac{t}{2}\right)\right). \end{aligned}$$

Thus, for all $t > 0$, we have

$$M(Tp^*, p^*, t) \geq \Delta\left(M\left(Tp^*, p_{n-1}, \frac{t}{2}\right), M\left(p^*, p_n, \frac{t}{2}\right)\right) = 1.$$

Therefore, p^* is the unique fixed point of T . This achieves the proof.

Theorem 3.3. Let (X, M, \min) be a complete fuzzy metric space with continuous t-norm \min .

Let $T : X \rightarrow X$ be a contraction mapping satisfying condition (1). Then T has a fixed point in X .

Proof. Let $p_0 \in X$ and let $\{p_n\}$ be the sequence of iterates of p_0 defined by $p_n = Tp_{n-1}$, $n = 1, 2, \dots$. Then for every positive integer m ,

$$(1 - \mu)(\mu + \mu^2 + \dots + \mu^m) = \mu - \mu^{m+1} < 1,$$

whence we have

$$\begin{aligned} M(p_0, p_m, t) &\geq M(p_0, p_m, (1 - \mu)(\mu + \mu^2 + \dots + \mu^m)t) \\ &\geq \min(M(p_0, p_m, (1 - \mu)t), \dots, M(p_{m-1}, p_m, (1 - \mu)\mu^{m-1}t)) \\ &\geq M(p_0, p_1, (1 - \mu)t) \end{aligned}$$

Thus $G_{p_0}M(p_0, p_1, (1 - \mu)I)$, where $I: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is the identity function. Therefore, the conclusion now follows from Lemma 3.2. This achieves the proof.

IV. HICKS FIXED POINT THEOREM AND ITS GENERALIZATION

In 1983, Hicks [3] introduced another notion of contraction mappings, which refer to as an contraction. In this section, we present the version of H-contraction in fuzzy metric space.

Definition 4.1. Let (X, M, Δ) be a fuzzy metric space with continuous t -norm Δ . A mapping

$T : X \rightarrow X$ is called H -contraction mapping if for any $t > 0$ and for any $x, y \in X$,

$$M(Tx, Ty, kt) > 1 - kt \quad (2)$$

whenever $M(x, y, t) > 1 - t$, where $k \in (0, 1)$ is a constant.

Lemma 4.2. Let (X, M, Δ) be a complete fuzzy metric space with continuous t -norm Δ , $T : X \rightarrow X$ be H -contraction mapping satisfying (2),

- 1) For any $\varepsilon > 0, \lambda > 0$, there exists a positive integer $n(\varepsilon, \lambda)$ such that for all $p, q \in X$ and

$$n \geq n(\varepsilon, \lambda),$$

$$M(T^n p, T^m q, \varepsilon) > 1 - \lambda, \quad (3)$$

- 2) T has at most one fixed point in X ,

- 3) T is uniformly continuous.

Proof. (1) Since clearly $M(p, q, 1 + \varepsilon) > 1 - (1 + \varepsilon)$, it follows from (2) that

$$M(T^n p, T^m q, k^n(1 + \varepsilon)) > 1 - k^n(1 + \varepsilon).$$

Since $k \in (0, 1)$ for any $\varepsilon > 0, \lambda \in (0, 1)$, there exists $n(\varepsilon, \lambda) \in \mathbb{N}$ such that $k^n(1 + \varepsilon) \leq \min\{\varepsilon, \lambda\}$ for any $n \geq n(\varepsilon, \lambda)$. Hence for any $n \geq n(\varepsilon, \lambda)$, we have

$$M(T^n p, T^m q, \varepsilon) \geq M(T^n p, T^m q, k^n(1 + \varepsilon)) > 1 - k^n(1 + \varepsilon) \geq 1 - \lambda.$$

(2) If $p, q \in X$ are fixed points of T , then for any $n \geq 1$ and $T^n p = p, T^n q = q$. By the conclusion (1), for any $\varepsilon > 0$ and for any $\lambda \in (0, 1)$, $M(p, q, \varepsilon) > 1 - \lambda$. This implies that $p = q$.

(3) Let $\varepsilon > 0$ and $\lambda \in (0, 1)$ be given and choose $\delta > 0$ such that $k\delta < \min\{\varepsilon, \lambda\}$. Now if $p, q \in N(\delta, \delta)$, where $N(\delta, \delta)$ is the (δ, δ) -neighborhood, i.e., $M(p, q, \delta) > 1 - \delta$. Since T is a H -contraction mapping, we have $M(Tp, Tq, k\delta) > 1 - \delta$. Hence, it follows that

$$M(Tp, Tq, \varepsilon) \geq M(Tp, Tq, k\delta) > 1 - k\delta > 1 - \delta,$$

which means that $(Tp, Tq) \in N(\varepsilon, \lambda)$. This achieves the proof.

Theorem 4.3. Let (X, M, Δ) be a complete fuzzy metric space with continuous t -norm Δ such that $\sup_{0 < t < 1} \Delta(t, t) = 1$. Then each H -contraction mapping T on X has a unique fixed point and, for any $p_0 \in X$, the iterative sequence $\{T^n p_0\}$ converges to this fixed point.

Proof. Let $\varepsilon > 0$ and $\lambda \in (0, 1)$ be given. By Lemma 4.2 (1) there exists a positive integer $n(\varepsilon, \lambda)$ such that (3) holds. Taking $p = p_m$ and $q = p_0$, then for all $n \geq n(\varepsilon, \lambda)$ and $m \geq 1$, we have

$$M(p_{n+m}, p_n, \varepsilon) = M(T^n p_m, T^n p_0, \varepsilon) > 1 - \varepsilon.$$

Therefore, $\{p_n\}$ is a Cauchy sequence. Since X is complete, we may assume that $p_n \rightarrow p^*$. By Lemma 4.2, it follows that $Tp^* = p^*$, i.e., p^* is a fixed point of T and it is unique. This achieves the proof.

If the t -norm Δ in Theorem 2 satisfies the following condition:

$$\Delta(a, b) \geq \max\{a + b - 1, 0\}, a, b \in [0, 1], \quad (4)$$

then we have the following:

Theorem 4.4. Let (X, M, Δ) be a complete fuzzy metric space with continuous t -norm Δ satisfying (4). Then

$$a) \quad d^*(x, y) = \sup\{t \in \mathbb{R}^+ : M(x, y, t) \leq 1 - t\} \quad (5)$$

is a metric on X , and the metric topology on X induced by d^* coincides with the topology τ on X induced by the family of neighborhoods:

$$\{U \subset X : \text{for each } x \in X, \text{ there exists } \varepsilon > 0 \text{ such that } N_x(\varepsilon, \varepsilon) \subset U\}, \quad (6)$$

where $N_x(\varepsilon, \varepsilon) = \{y \in X : M(x, y, \varepsilon) > 1 - \varepsilon\}$.

- b) The mapping $T: X \rightarrow X$ is an H -contraction on (X, M, Δ) if and only if T is a Banach contraction mapping on the metric space (X, d^*) , if and only if there exists a $k \in (0, 1)$ such that

$$d^*(Tx, Ty) \leq kd^*(x, y) \quad (7)$$

Proof. (a) First we point out from the definition of d^* defined by (5) has the following property:

$$d^*(x, y) < t \Leftrightarrow M(x, y, t) > 1 - t, \quad t > 0 \quad (8)$$

Now we prove that d^* is a metric on X . In fact, it is obvious that $d^*(x, y) \geq 0$, $d^*(x, y) = d^*(y, x)$

and $d^*(x, y) = 0$ if and only if $x = y$. Besides, by the definition of d^* , for any $\varepsilon > 0$ and $x, y, z \in X$, we have

$$\begin{aligned} M\left(x, z, d^*(x, z) + \frac{\varepsilon}{2}\right) &> 1 - d^*(x, z) - \frac{\varepsilon}{2}, \\ M\left(y, z, d^*(y, z) + \frac{\varepsilon}{2}\right) &> 1 - d^*(y, z) - \frac{\varepsilon}{2}. \end{aligned} \quad (9)$$

Hence from the definition of FM-space and the above expression it follows that

$$\begin{aligned} M(x, y, d^*(x, z) + d^*(y, z) + \varepsilon) &\geq \Delta\left(M\left(x, z, d^*(x, z) + \frac{\varepsilon}{2}\right), M\left(y, z, d^*(y, z) + \frac{\varepsilon}{2}\right)\right) \\ &\geq M\left(x, z, d^*(x, z) + \frac{\varepsilon}{2}\right) + M\left(y, z, d^*(y, z) + \frac{\varepsilon}{2}\right) - 1 \\ &\geq 1 - (d^*(x, y) + d^*(y, z) + \varepsilon). \end{aligned}$$

Using (8), we have

$$d^*(x, y) < d^*(x, z) + d^*(z, y) + \varepsilon$$

Letting $\varepsilon \rightarrow 0$, we have

$$d^*(x, y) \leq d^*(x, z) + d^*(z, y), \text{ for all } x, y, z \in X.$$

Next, in order to prove the metric topology induced by d^* coincides with the topology τ induced by the family of neighborhoods defined by (6), it suffices to note from (8) we can prove that

$$N_x(\varepsilon, t) = \{y \in X: d^*(x, y) < \varepsilon\}$$

This achieves the proof of (a).

- (b) If $T: X \rightarrow X$ is an H -contraction on (X, M, Δ) , then for any $t > 0$ such that $M(x, y, t) > 1 - t$, we have

$$M(Tx, Ty, kt) > 1 - kt.$$

By (8), that is to say, for any $t > 0$, if $d^*(x, y) < t$, then we have $d^*(Tx, Ty) < kt$. Letting $t \rightarrow d^*(x, y)$ for all $x, y \in X$, we have

$$d^*(Tx, Ty) \leq kd^*(x, y) \quad (10)$$

This shows that T is a Banach contraction mapping on (X, d^*) . Conversely, if T is a Banach contraction mapping on (X, d^*) satisfying (10), then for any $t > 0$ such that $M(x, y, t) > 1 - t$. By (8), we have $d^*(x, y) < t$. From (10), it follows that $d^*(Tx, Ty) < kt$. Hence

$$M(Tx, Ty, kt) > 1 - kt.$$

which shows that T is a H -contraction. This achieves the proof of (b).

V. COMPARISON OF **SB**-CONTRACTION AND **H**-CONTRACTION

It follows at once from Theorem 4.3 that every H -contraction on a complete fuzzy metric space (X, M, Δ) with $\Delta \geq \Delta_m$ has a unique fixed point. But this not the case for SB -contraction. Thus a SB -contraction need not be an H -contraction. Similarly, as the following shows, an H -contraction need not be a SB -contraction.

Example 5.1. Let $E = \mathbb{N} \cup \{0\}$ and for any $p, q \in E$, define

$$M(x, y, t) = \begin{cases} 0, & \text{if } t \leq 2^{-\min\{p, q\}}; \\ 1 - 2^{-\min\{p, q\}}, & \text{if } 2^{-\min\{p, q\}} < t \leq 1; \\ 1, & \text{if } t > 1. \end{cases}$$

It is straightforward to verify that (X, M, \min) is a fuzzy metric space. Define $T : E \rightarrow E$ by $T(n) = n + 1$. Since $\min \geq \Delta_m$ and

$$\sup_t \{t \in \mathbb{R}^+ : M(Tx, Ty, t) \leq 1 - t\} = \frac{1}{2} \sup_t \{t \in \mathbb{R}^+ : M(x, y, t) \leq 1 - t\},$$

therefore, if $t > 0$ and satisfies $M(x, y, t) > 1 - t$, then we have $M(Tx, Ty, \frac{1}{2}t) > 1 - \frac{1}{2}t$. This means that T is a H -contraction mapping on (E, M, \min) . Next, let $r \in (0, 1)$ be any number and choose $t \in \frac{1}{r}$. Then $rt < 1$, so that

$$M(T(0), T(1), rt) = M(1, 2, rt) \leq \frac{1}{2} < 1 = M(0, 1, t).$$

Thus T is a SB -contraction mapping. The above discussion shows in general the H -contraction and SB -contraction are independent.

Now, the following lemma explains that that SB -contraction in a fuzzy metric space is stronger than that of H -contraction.

Lemma 5.2. Let (X, M, Δ) be a fuzzy metric space. If T is SB -contraction and if $M(Tx, Ty, \cdot)$ is strictly increasing on $[0, 1]$, then $\beta(Tx, Ty) < \beta(x, y)$, where

$$\beta(x, y) = \inf\{t : M(x, y, t) > 1 - t\}.$$

Proof. We find η such that $0 < \eta < \frac{1-\gamma}{\gamma} \beta(x, y)$. Then, we have $\beta(x, y) > \gamma [\beta(x, y) + \eta]$. Since

$M(Tx, Ty, \cdot)$ is strictly increasing on $[0, 1]$, so $0 \leq \beta(x, y) \leq 1$, and since T is a SB -contraction, so we have

$$\begin{aligned} \beta(Tx, Ty) &= M(Tx, Ty, \beta(x, y)) > M(Tx, Ty, \gamma [\beta(x, y) + \eta]) \\ &\geq M(x, y, \beta(x, y)) + \eta > 1 - \beta(x, y) \end{aligned}$$

This implies that $\beta(Tx, Ty) < \beta(x, y)$. This achieves the proof.

In general, every SB -contraction need not be an H -contraction. To show this, we have the following example:

Example 5.3. For each integer n , let $p_n : (0, 1) \rightarrow \mathbb{R}^+$ be given by $p_n(t) = 2^{-n}(1 - t)t^{-1}$. Also, let $X = \{p_n : n \text{ is an integer}\}$, let P be Lebesgue measure on $(0, 1)$, and for $x \geq 0$, let

$$M(p_n, p_m, x) = P\{t \in (0, 1) : |p_n(t) - p_m(t)| < x\} = \frac{x}{x + |2^{-n} - 2^{-m}|}.$$

Then, a function $T : X \rightarrow X$ defined by $T(p_n) = p_{n+1}$ is a SB -contraction but not is H -contraction.

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