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PID Augmented Stability of Multi–Rotor Helicopters

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***Abstract*- This paper is a general theoretical formalization of the design of linear controllers for flight stabilization of multi–rotor helicopters. After the description of the linear dynamic modelling of this type of aircraft, the plants transfer functions are obtained for the sizing of *PID* regulators. The gains of the controllers that assure the dynamic stability of the system, in a purely analytical way, are then computed. These gains are given as functions of the aerodynamic and mechanical features of the aircraft. All in closed form expressions, these results represent the first exact solution to the problem of stabilization of attitude and velocity of this type of aircraft, through linear *PID* control. Affordable tools useful to a quick hand calculation for control systems tuning, dynamic stability evaluation, multi–rotor aircraft design and also for academic educational purposes are thus provided and then tested in a numeric simulation.**

Keywords: Multi–Rotor, Helicopter, *PID*, Flight Stabilization, Linear Modelling

I. Introduction

Unmanned multi–rotor flying helicopters are a notable subject of recent research activities in academic institutions [1, 2, 3, 4, 5] and of relevant commercial and hobby applications.

Multi–rotor aircraft are rotorcraft able to fly, hover, take off and land vertically. Their flight capabilities are possible thanks to their electric driven propellers. The propellers grant indeed sustenance, thrust and control actions. In almost the totality of vehicles all the input actions are imparted through variations of the spin rates of the rotors [1]. The most widespread and typical applications feature 4 (quad–copter or quad–rotor), 6 (hexa–copter) or 8 rotors (okta–copter). The great number of installed propellers allows the aircraft to be perfectly balanced during flight in front of the aerodynamic torques due to the rotors themselves.

All these vehicles, generally, necessitate artificial stabilization devices to fly safely. Thence, efforts of research are aimed to the design of their control systems. To do that, different methodologies have been used. Beyond classical linear controllers, like *PID* and *LQ* regulators [6, 7, 8], other sophisticated solutions were considered [2, 9]. These involve backstepping control, full state feedback control, fuzzy control and other non–linear control techniques [10, 11, 12].

Despite these last options, in the more widespread case of hobby and commercial aircraft, the stabilization is accomplished through linear *PID* regulators [13], due to their relative ease of implementation and robustness [14, 15, 16]. Till now, however, in the published works on the subject, the sizing of this type of controllers is made often with not complete attention to the inherent dynamic stability of the flying system itself, despite the notable depth reached in the math modelling of flight dynamics of these machines. Sometimes the tuning is done with 'trial–and–error' attempts [17] or, alternatively, with artificial numerical optimization processes,

G, G_c, H	Plant, controller and transducer transfer functions
h, b	Height and arm length of rotors
I_{xx}, I_{yy}, I_{zz}	Inertia moments
L_v, L_p, M_u, M_q, N_r	Rotational stability derivatives
L_{lat}, M_{lon}, N_{lon}, N_{lat}, N_{rud}	Rotational control derivatives
m	Mass
μ_z, λ_i	Climb ratio and inflow ratio of a rotor
N	Rotor blades number
N_{rot}	Number of rotors of the aircraft
Ω_j	Spin rate of te j–th rotor
ϕ, θ, ψ	Roll, pitch and heading angles
p, q, r	Roll, pitch and yaw angular rates
ρ	Air density
s	Complex variable
$\mathbf{T}_j$	Rotation matrix of the j–th rotor
t	Time variable
θ_c, θ_{tw}	Collective pitch and linear twist of rotors blades
u_{col}, u_{lon}, u_{lat}, u_{rud}	Input vector components
u, v, w	Velocity components in body axis
$\mathbf{x}$, $\mathbf{u}$	State and input vectors
X_u, X_q, Y_p, Y_v, Z_w	Translational stability derivatives
X_{lon}, Y_{lat}, Z_{col}	Translational control derivatives

Appendix: Stability and Control Derivatives of a Hovering Multi–Rotor Helicopter

The stability and control derivatives contain the effects of aerodynamics of rotors, rotors displacement and orientation, mass and mass distribution of the aircraft.

All the aerodynamic perturbations are included in the derivatives of C_T and C_Q of all the rotors. In the case of hovering flight the unique perturbation effect on C_T and C_Q is that due to the climb ratio μ_z of any propeller [29].

$$X_u = -\frac{1}{m}\sum_{j=1}^{N_{rot}} \frac{\partial C_T}{\partial \mu_z}\ \rho A\ \Omega_0\ R\ \mathbf{T}_j(3,1)^2 \tag{47}$$

$$X_q = -\frac{1}{m}\sum_{j=1}^{N_{rot}} \frac{\partial C_T}{\partial \mu_z}\ \rho A\ \Omega_0\ R\ \mathbf{T}_j(3,1)\left[(-b\cos\delta_j)\ \mathbf{T}_j(3,3) - h\ \mathbf{T}_j(3,1)\right] \tag{48}$$

$$Z_w = -\frac{1}{m}\sum_{j=1}^{N_{rot}} \frac{\partial C_T}{\partial \mu_z}\ \rho A\ \Omega_0\ R\ \mathbf{T}_j(3,3)^2 \tag{49}$$

$$M_u = -\frac{1}{I_{yy}}\sum_{j=1}^{N_{rot}} \frac{\partial C_T}{\partial \mu_z}\ \rho A\ \Omega_0\ R\ \mathbf{T}_j(3,1)\left[h\mathbf{T}_j(3,1) + (-b\cos(\delta_j))\mathbf{T}_j(3,3)\right] \tag{50}$$

$$M_q = -\frac{1}{I_{yy}}\sum_{j=1}^{N_{rot}} \frac{\partial C_T}{\partial \mu_z}\ \rho A\ \Omega_0\ R\left[b\cos(\delta_j)\mathbf{T}_j(3,3) + (-h)\mathbf{T}_j(3,1)\right]^2 \tag{51}$$

$$\begin{aligned} N_r = & -\frac{1}{I_{zz}}\sum_{j=1}^{N_{rot}} \frac{\partial C_T}{\partial \mu_z}\ \rho A\ \Omega_0\ R\ b^2\tilde{\mathbf{T}}_j(3,2)^2 + \\ & +\frac{1}{I_{zz}}\sum_{j=1}^{N_{rot}} \frac{\partial C_Q}{\partial \mu_z}\ \rho A\ \Omega_0\ sign(\Omega_{0,j})\ R^2\ b\tilde{\mathbf{T}}_j(3,2)\tilde{\mathbf{T}}_j(3,3) \end{aligned} \tag{52}$$

$$Z_{col} = \frac{1}{m}\sum_{j=1}^{N_{rot}} \mathbf{T}_j(3,3)\left[-\frac{mg}{N_{rot}\cos(\Gamma_j)\cos(\xi_j)}\frac{2}{\Omega_0}\right] \tag{53}$$

$$\begin{aligned} M_{lon} = & \frac{1}{I_{yy}}\sum_{j=1}^{N_{rot}} \frac{mg}{N_{rot}\cos(\Gamma_j)\cos(\xi_j)}\frac{2}{\Omega_0}\cdot \\ & \cdot\left[-\mathbf{T}_j(3,3)(-b)\cos(\delta_j)\ sign\{-\cos(\delta_j)\} - \mathbf{T}_j(3,1)\ h\ sign\{-\cos(\delta_j)\}\right] \end{aligned} \tag{54}$$

$$\begin{aligned} N_{rud} = & \frac{1}{I_{zz}}b\sum_{j=1}^{N_{rot}}\left[-\tilde{\mathbf{T}}_j(3,2)\frac{mg}{N_{rot}\cos(\Gamma_j)\cos(\xi_j)}\frac{2}{\Omega_0}\ sign\{\xi_j\}\right] + \\ & +\frac{1}{I_{zz}}\sum_{j=1}^{N_{rot}}\left[\tilde{\mathbf{T}}_j(3,3)\ C_{Q0,j}\ \rho\ A\ 2\Omega_0\ R^3\right] \end{aligned} \tag{55}$$

$\mathbf{T}_j$ is the rotation matrix of the $j-$th rotor, defined by the three angle δ_j, Γ_j and ξ_j [29]. This matrix describes the orientation of the $j-$th rotor disk with respect to the body axis frame of the whole machine. δ_j defines the azimuth displacement of each rotor. Γ_j is the dihedral angle that forces each rotor to direct a component of its thrust along its arm. ξ_j is the third angle that completes, for any rotor, the set of Euler's angles. For torque balancing, Γ_j must be equal for all rotors and two adjacent rotors must possess opposite values of ξ_j.

$\tilde{\mathbf{T}}_j$ is the rotation matrix of the $j-$th rotor with $\delta_j = 0^o$.

Ω_0 is the magnitude of the rotors spin rates $\Omega_{0,j}$. In hovering flight Ω_0 is the same for all the rotors.

$$\Omega_0 = \frac{1}{4R}\sqrt{\frac{T_0}{2\rho A}\left[\frac{1+\sqrt{1+\frac{64}{\sigma C_{l\alpha}}\left(\frac{\theta_c}{3}-\frac{\theta_{tw}}{4}\right)}}{\frac{\theta_c}{3}-\frac{\theta_{tw}}{4}}\right]} \tag{56}$$

In the previous equation T_0 is the hovering rotor thrust.

$$T_0 = \frac{mg}{N_{rot}\cos(\xi_j)\cos(\Gamma_j)} \tag{57}$$

λ_{i0} is the inflow ratio of a rotor in hovering flight.

$$\lambda_{i0} = \sqrt{\frac{T_0}{2\rho A(\Omega_0 R)^2}} \tag{58}$$

$$\frac{\partial C_T}{\partial \mu_z} = \frac{2\sigma C_{L\alpha}\lambda_{i0}}{16\lambda_{i0} + C_{L\alpha}\sigma} \tag{59}$$

$$\frac{\partial C_Q}{\partial \mu_z} = \frac{-4\sigma C_{L\alpha}}{16\lambda_{i0} + \sigma C_{L\alpha}}\left(\frac{\theta_c}{3} - \frac{\theta_{tw}}{4} - \lambda_{i0}\right) \tag{60}$$

From rotors aerodynamics theory the next results, in the case of hovering flight, descend.

$$\begin{cases} \frac{\partial C_T}{\partial \mu_z} > 0 \\ \frac{\partial C_Q}{\partial \mu_z} < 0 \end{cases} \tag{61}$$

Typically it is also that $|\partial C_Q/\partial \mu_z| << |\partial C_T/\partial \mu_z|$.

$C_{Q0,j}$ is the torque coefficient in hovering flight for the $j-$th rotor.

$$C_{Q0,j} = \left[\sigma C_{l\alpha}\left(\frac{\theta_c}{6} - \frac{\theta_{tw}}{8} - \frac{\lambda_{i0}}{4}\right)\lambda_{i0} + \frac{\sigma C_d}{8}\right] \tag{62}$$

The control derivatives are computed after the hypothesis that the C_T and C_Q remain constant in case of a small variation of the rotor spin rate.

With all the previous considerations the signs of the derivatives can be checked.

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